

## Note on CKM matrix renormalization

Yi Liao

Institut für Theoretische Physik, Universität Leipzig,  
Augustusplatz 10/11, D-04109 Leipzig, Germany

### Abstract

A simple inspection of the one loop quark self-energy suggests a prescription of the CKM matrix renormalization in the standard model. It leads to a CKM matrix counterterm which is gauge parameter independent and satisfies the unitarity constraint, and renormalized physical amplitudes which are gauge parameter independent and smooth in quark mass difference. We make a point that caution should be practiced when interpreting the CKM matrix counterterm in terms of those of parameters in a given representation due to rephasing effects from renormalization. We show how this can be done using the degrees of freedom in the on-shell renormalization scheme.

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# 1 Introduction

The Cabibbo-Kobayashi-Maskawa (CKM) matrix appearing in the charged current sector of the standard model (SM) arises from a mismatch in the transformations of the up-type and down-type quark fields that bring them from weak gauge eigenstates to mass eigenstates [1]. As the matrix contains free physical parameters, it will generally be subject to renormalization. Concerning renormalization, there is an essential difference between these parameters and other physical parameters in SM. The latter parameters include the masses of all physical particles and the fine structure constant. There is a physically natural way to define or renormalize them: the mass of a physical particle can be identified with the real part of the pole of the corresponding field propagator; and the fine structure constant can be defined in terms of the Thomson cross section due to a theorem which states that the cross section of a soft photon scattering against a massive particle approaches its classical result in the low energy limit. In contrast, it does not make much sense to speak of gauge eigenstates beyond the Lagrangian level. There is thus no physically preferred way to define the CKM matrix at higher orders.

The necessity to renormalize the CKM matrix in order to obtain an ultraviolet (UV) finite result for a physical amplitude was first analyzed by Marciano and Sirlin for two generations [2]. The case of three generations was then studied by Denner and Sack [3] in the on-shell renormalization scheme of SM [4]. A prescription was proposed for the counterterm of the matrix, which is a combination of the quark wave-function renormalization constants specified in the on-shell scheme. Unfortunately, the counterterm so determined turns out to contain a UV finite part that is gauge parameter dependent [5], which should be avoided for physical parameters. An alternative prescription was then suggested [5, 6], which is based on the quark wave-function renormalization constants determined at zero momentum and shown to be independent of gauge parameter. Such a prescription necessarily departs from the on-shell renormalization scheme. To work exclusively in terms of on-shell renormalization constants, the authors of Ref. [7] proposed to renormalize the matrix with respect to a reference theory in which no mixing occurs. The program was further improved and elaborated upon in Ref. [8]. Yet another approach [9] was developed on the pinch technique that is often used to tackle the problem of gauge parameter dependence. From that point of view, the original prescription of Denner and Sack may be reinterpreted as one of many possible ways to separate out a counterterm for the CKM matrix that is gauge parameter independent, and is thus acceptable. All of these ap-

proaches thus differ only in a UV finite and gauge parameter independent part in the counterterm for the CKM matrix and are understood as renormalization scheme dependence in Ref. [10]. The issue has also been investigated in relation to the wave-function renormalization for unstable particles [11, 12].

In this work we will show how a simple inspection of the one loop contribution to quark self-energies suggests a way of splitting them: one part that is UV divergent but gauge parameter independent, to be absorbed into the CKM matrix counterterm, and the other that is UV finite but gauge parameter dependent, to be put back to form renormalized physical amplitudes. The CKM matrix counterterm so obtained shares with all proposals made so far the requisite properties: gauge parameter independence, unitarity constraints and absorption of the remaining UV divergence in physical amplitudes. It also enjoys a nice feature that is incorporated in the prescriptions of Refs. [3, 7, 8, 9]; namely, the renormalized physical amplitudes are smooth when the up-type (or down-type) quark masses approach each other. The result is similar to that of Ref. [9] and differs in the UV finite and gauge parameter independent terms, while avoiding the heavy machinery of the pinch technique. We will also make a point that seems to have not been emphasized in the literature. When one works in a specific representation of the CKM matrix which is often convenient in practical calculations, caution must be exercised in interpreting the CKM matrix counterterm in terms of its rotation angles and CP phase. There is a relative rephasing, i.e., a change of representations between the bare and renormalized CKM matrices due to renormalization effects. This rephasing must be removed before one can write down the counterterms for those angles and phase. We show how this can be done using the degrees of freedom available in the on-shell renormalization scheme.

## 2 Renormalization of the CKM Matrix

As the  $Wud$ -type vertex is the only available interaction term among physical particles that involves the CKM matrix, it is natural to use it as a reference to renormalize the CKM matrix. The one-loop renormalized amplitude for the decay  $W^+ \rightarrow u_\alpha \bar{d}_i$  is [3]

$$\begin{aligned} \mathcal{A} = & -\frac{e}{\sqrt{2}s_W} \bar{u}_\alpha \not{\epsilon} P_L v_i \left[ \left( \frac{1}{2} \sum_\beta \delta U_{\beta\alpha}^{L*} V_{\beta i} + \frac{1}{2} \sum_j V_{\alpha j} \delta D_{ji}^L + \delta V_{\alpha i} \right) + V_{\alpha i} (1 + \delta C) \right] \\ & + \text{other terms.} \end{aligned} \quad (1)$$

The up-type and down-type quarks are distinguished by the greek and italic letters, respectively, so that the  $(\alpha, i)$  entry of the CKM matrix is denoted as  $V_{\alpha i}$  with the counterterm

$\delta V_{\alpha i}$ .  $\epsilon$  is the polarization vector of the  $W^+$  boson and  $P_L = (1 - \gamma_5)/2$ ,  $P_R = (1 + \gamma_5)/2$ . We shall work throughout in the on-shell renormalization scheme.  $U^{L,R} = 1 + \delta U^{L,R}$  and  $D^{L,R} = 1 + \delta D^{L,R}$  are, respectively, the wave-function renormalization constant matrices for the left- or right-handed up-type and down-type quark fields. When distinction over up- and down-type quarks is not necessary, we use  $Z^{L,R} = 1 + \delta Z^{L,R}$  instead and the italic letters for the flavors. In the above formula,  $\delta C = e^{-1} \delta e - s_W^{-1} \delta s_W + 1/2 \delta Z_W + C_{1\text{PI}}$ . Here  $\delta e$ ,  $\delta s_W$ , and  $Z_W = 1 + \delta Z_W$  are, respectively, the counterterms for the electromagnetic coupling, the sine of the weak mixing angle and the wave-function renormalization constant for the  $W$  boson, while  $C_{1\text{PI}}$  stands for the on-shell 1PI vertex contribution which shares the same Lorentz structure as the tree level amplitude. It is sufficient for us to know that without  $\delta V_{\alpha i}$  the quantity in the above square parentheses is gauge parameter independent but UV divergent. The remaining UV divergences are expected to be cancelled by  $\delta V_{\alpha i}$ , which in turn must meet at least two more conditions. First, it must satisfy  $\delta V^\dagger V + V^\dagger \delta V = 0$  to guarantee the unitarity of both bare and renormalized matrices so that the number of independent physical parameters is not changed by renormalization. Second, it does not introduce new gauge parameter dependence. The other terms not explicitly displayed in the above formula are the on-shell 1PI vertex contributions of different Lorentz structures. These terms are separately UV finite and gauge parameter independent as they must, and are thus of no concern for our analysis.

Let us first review how wave-function renormalization constants and mass counterterms are determined in the on-shell renormalization scheme in the presence of mixing [4]. We denote the renormalized mixing or self-energy for the transition  $j \rightarrow k$  as  $\Gamma_{kj}(p)$ . For the purpose of determining the above mentioned counterterms, only the dispersive part in  $\Gamma_{kj}(p)$  is retained. This will always be implied in the following discussion. The Hermiticity of the effective action then demands that  $\gamma_0 \Gamma^\dagger \gamma_0 = \Gamma$ . Thus, it can be parametrized as

$$\Gamma_{kj}(p) = \not{p} P_L F_{kj}^L(p^2) + \not{p} P_R F_{kj}^R(p^2) + P_L F_{kj}^S(p^2) + P_R F_{jk}^{S*}(p^2), \quad (2)$$

with  $F_{kj}^{L*} = F_{jk}^L$ ,  $F_{kj}^{R*} = F_{jk}^R$ . The on-shell renormalization conditions that the renormalized mass is identified with the zero of the real part of the self-energy, that no mixing occurs between two particles when either of them is on-shell, and that the residue of the diagonal propagator at the pole is unity, are equivalent to the following equations,

$$\Gamma_{kj}(p) u_j(p) |_{p^2 \rightarrow m_j^2} = 0,$$

$$\frac{1}{\not{p} - m_j} \Gamma_{jj}(p) u_j(p) \big|_{p^2 \rightarrow m_j^2} = 1. \quad (3)$$

At one loop level,  $\Gamma_{kj}(p) = (\not{p} - m_j) \delta_{kj} + \Gamma_{kj}^{\text{loop}}(p) + \Gamma_{kj}^{\text{ct}}(p)$ . Since the counterterm contribution  $\Gamma_{kj}^{\text{ct}}(p)$  fulfills the Hermitian property separately, the latter must also be respected by the loop contribution.  $\Gamma_{kj}^{\text{loop}}(p)$  can be parametrized as in Eq. (2) with  $F$ 's replaced by  $\Sigma$ 's. We then have  $\Sigma_{kj}^{L\star} = \Sigma_{jk}^L$ ,  $\Sigma_{kj}^{R\star} = \Sigma_{jk}^R$ .

The first condition in Eq. (3) yields for  $j \neq k$ ,

$$\begin{aligned} \delta Z_{kj}^L &= \frac{2}{m_k^2 - m_j^2} \left[ m_j^2 \Sigma_{kj}^L + m_j m_k \Sigma_{kj}^R + m_k \Sigma_{kj}^S + m_j \Sigma_{jk}^{S\star} \right] (m_j^2), \\ \delta Z_{kj}^R &= \frac{2}{m_k^2 - m_j^2} \left[ m_j^2 \Sigma_{kj}^R + m_j m_k \Sigma_{kj}^L + m_j \Sigma_{kj}^S + m_k \Sigma_{jk}^{S\star} \right] (m_j^2), \end{aligned} \quad (4)$$

and for  $j = k$ ,

$$\begin{aligned} \delta m_j &= \frac{1}{2} \left[ m_j (\Sigma_{jj}^L + \Sigma_{jj}^R) + \Sigma_{jj}^S + \Sigma_{jj}^{S\star} \right] (m_j^2), \\ \delta Z_{jj}^L - \delta Z_{jj}^R &= \left[ \Sigma_{jj}^R - \Sigma_{jj}^L + \frac{\Sigma_{jj}^S - \Sigma_{jj}^{S\star}}{m_j} \right] (m_j^2). \end{aligned} \quad (5)$$

The above diagonal equations are also covered by the second condition in Eq. (3) using the Hermitian property. In addition, the condition also yields the following results:

$$\begin{aligned} \text{Re } \delta Z_{jj}^L &= -\Sigma_{jj}^L(m_j^2) - m_j \left[ m_j (\Sigma_{jj}^{L'} + \Sigma_{jj}^{R'}) + (\Sigma_{jj}^{S'} + \Sigma_{jj}^{S'\star}) \right] (m_j^2), \\ \text{Re } \delta Z_{jj}^R &= -\Sigma_{jj}^R(m_j^2) - m_j \left[ m_j (\Sigma_{jj}^{L'} + \Sigma_{jj}^{R'}) + (\Sigma_{jj}^{S'} + \Sigma_{jj}^{S'\star}) \right] (m_j^2), \end{aligned} \quad (6)$$

with  $\Sigma_{jj}^{L'}(m_j^2) = \frac{\partial}{\partial p^2} \Sigma_{jj}^L(p^2) \big|_{p^2=m_j^2}$  etc. The mass counterterms and the off-diagonal wavefunction renormalization constants are uniquely determined, while the diagonal ones are determined up to a difference in imaginary parts for each  $j$ ,

$$\text{Im } \delta Z_{jj}^L - \text{Im } \delta Z_{jj}^R = \frac{2}{m_j} \text{Im } \Sigma_{jj}^S(m_j^2). \quad (7)$$

When  $\text{Im } \Sigma_{jj}^S(m_j^2) = 0$ , as is the case at one loop in SM, we can choose arbitrarily a common imaginary part for  $\delta Z_{jj}^{L,R}$  for each  $j$ . This freedom is already contained in the on-shell renormalization scheme [4]: for a given set of  $Z^{L,R}$ , a common rephasing  $Z^{L,R} \rightarrow E Z^{L,R}$  with  $E = \text{diag}(e^{i\varphi_1}, e^{i\varphi_2}, \dots)$  does not leave any trace in the kinetic terms and the interaction terms in the neutral current sector which involve quark fields of the same type. The off-diagonal  $Z_{kj}^{L,R}$  starts at  $O(e^2)$  and can thus feel the arbitrariness only at two loop level, while the diagonal  $Z_{jj}^{L,R}$  starts at  $O(1)$  and the arbitrariness shows up already at one loop level. However, in the charged current sector where both types of

quarks participate, the rephasing does affect the appearance of the CKM matrix. When we are supposed to renormalize in a specific representation of the CKM matrix, this rephasing degree of freedom should be considered together with the CKM matrix renormalization. We will illustrate how this can be done later on.

A peculiar feature in the off-diagonal renormalization constants is that they become singular as the masses of the two quarks under consideration approach each other. If the degeneracy is exact, it must be protected by some symmetry from renormalization effects so that the two quarks decouple from mixing with other quarks of the same type. More interesting is the case when the mass difference of two quarks is much smaller than their mass scale. In such a case, there is no reason that the CKM matrix, as independent physical parameters, must be trivial with respect to these closely lying quarks of the same type. Although this does not occur phenomenologically in SM, it is theoretically natural to expect that physical amplitudes like Eq. (1) should be smooth in the mass difference. As the nonsmoothness is caused by the mixing, it should in physical amplitudes be reabsorbed into the counterterm for the mixing matrix. Now we show that this can be readily arranged.

The quark wave-function renormalization constants appearing in Eq. (1) can be decomposed as follows:

$$\begin{aligned}
& \frac{1}{2} \sum_{\beta} \delta U_{\beta\alpha}^{L*} V_{\beta i} + \frac{1}{2} \sum_j V_{\alpha j} \delta D_{ji}^L \\
&= \frac{1}{4} \left[ \sum_{\beta} (\delta U_{\beta\alpha}^{L*} + \delta U_{\alpha\beta}^L) V_{\beta i} + \sum_j V_{\alpha j} (\delta D_{ji}^L + \delta D_{ij}^{L*}) \right] \\
&+ \frac{1}{4} \left[ \sum_{\beta} (\delta U_{\beta\alpha}^{L*} - \delta U_{\alpha\beta}^L) V_{\beta i} + \sum_j V_{\alpha j} (\delta D_{ji}^L - \delta D_{ij}^{L*}) \right].
\end{aligned} \tag{8}$$

Consider first the Hermitian combination of  $\delta Z^L$ . Using Eq. (4) and the Hermitian property of  $\Sigma^{L,R}$ , we see that the off-diagonal terms are smooth in quark mass difference; the diagonal term is proportional to  $\text{Re } \delta Z_{jj}^L$  and free of the arbitrariness mentioned above. This part cannot enter into  $\delta V_{\alpha i}$  since it does not fulfil the unitarity constraint. In contrast, the anti-Hermitian combination does fulfil the constraint and is not smooth in mass difference. It should thus play a role in constructing  $\delta V_{\alpha i}$ . The diagonal term of the combination is

$$+\frac{i}{2} V_{\alpha i} \text{Im} \left[ -\delta U_{\alpha\alpha}^L + \delta D_{ii}^L \right], \tag{9}$$

which is not fixed in the on-shell scheme due to the arbitrariness. For the off-diagonal terms, we use Eq. (4) and the explicit one loop results listed in Ref. [6] for  $\Sigma^{L,R,S}$  in

general  $R_\xi$  gauge. Note that at this order only the charged current loops can contribute. Using the loop integrals  $B_0, B_1$  defined there and  $A(m) = m^2[B_0(0, m, m) + 1]$  and making free use of unitarity of  $V$ , the contribution in the up-type sector is arranged as follows:

$$\begin{aligned}
& \frac{1}{4} \sum_{\beta \neq \alpha} \left( \delta U_{\beta\alpha}^{L\star} - \delta U_{\alpha\beta}^L \right) V_{\beta i} \\
&= -\frac{\alpha}{8\pi s_W^2 m_W^2} \sum_{\beta \neq \alpha} \sum_j V_{\beta i} V_{\alpha j} V_{\beta j}^* \\
&\times \left\{ \frac{1}{4} \left[ (\xi_W m_W^2 + m_j^2 - m_\alpha^2) B_0(\alpha, j, \xi_W) - (\alpha \rightarrow \beta) \right] + \frac{1}{2} \frac{m_\beta^2 + m_\alpha^2}{m_\beta^2 - m_\alpha^2} A(j) \right. \\
&\quad \left. + \frac{1}{2} \frac{1}{m_\beta^2 - m_\alpha^2} \left[ m_\alpha^2 (m_W^2 B_1(\alpha, j, W) - (m_W^2 + m_j^2 - m_\alpha^2) B_1(\alpha, W, j)) + (\alpha \rightarrow \beta) \right] \right\}, \tag{10}
\end{aligned}$$

where we have used the abbreviations

$$\begin{aligned}
A(j) &= A(m_j), \\
B_1(\alpha, W, j) &= B_1(m_\alpha^2, m_W, m_j), \\
B_0(\alpha, j, \xi_W) &= B_0(m_\alpha^2, m_j, \sqrt{\xi_W} m_W),
\end{aligned}$$

etc. The second and third terms in the above are singular in mass difference, UV divergent but  $\xi_W$  independent, while the first one is smooth in mass difference, UV finite upon summation over  $j$  but contains a  $\xi_W$ -dependent UV finite part. It is thus natural to absorb the second and third terms into  $\delta V_{\alpha i}$  and put the first term together with the Hermitian combination back into Eq. (1) to form the renormalized amplitude. Such a rearrangement meets all requisite conditions and also incorporates the smoothness property into physical amplitudes. Including the down-type part, we propose the following counterterm for the CKM matrix:

$$\begin{aligned}
& \left( \frac{\alpha}{16\pi s_W^2 m_W^2} \right)^{-1} \tilde{\delta} V_{\alpha i} \\
&= \sum_{\beta \neq \alpha} \sum_j \frac{V_{\beta i} V_{\alpha j} V_{\beta j}^*}{m_\beta^2 - m_\alpha^2} \left\{ (m_\beta^2 + m_\alpha^2) A(j) \right. \\
&\quad \left. + \left[ m_\alpha^2 (m_W^2 B_1(\alpha, j, W) - (m_W^2 + m_j^2 - m_\alpha^2) B_1(\alpha, W, j)) + (\alpha \rightarrow \beta) \right] \right\} \\
&+ \sum_{j \neq i} \sum_\beta \frac{V_{\beta i} V_{\alpha j} V_{\beta j}^*}{m_j^2 - m_i^2} \left\{ (m_j^2 + m_i^2) A(\beta) \right. \\
&\quad \left. + \left[ m_i^2 (m_W^2 B_1(i, \beta, W) - (m_W^2 + m_\beta^2 - m_i^2) B_1(i, W, \beta)) + (i \rightarrow j) \right] \right\}. \tag{11}
\end{aligned}$$

Let us compare  $\tilde{\delta} V$  with the prescriptions in Refs. [3, 9]. Denner and Sack proposed to absorb completely into  $\delta V$  the anti-Hermitian combination in Eq. (8) evaluated in the 't Hooft-Feynman gauge ( $\xi_W = 1$ ). This introduces UV finite  $\xi_W$  dependence into

$\delta V$  and thus also the renormalized amplitude [5], as shown here in the first term of Eq. (10). Yamada applied the pinch technique to guide the splitting of the quark wave-function renormalization constants. As the  $\xi_W = 1$  gauge serves as a reference point in the technique, the part split into the renormalized amplitude vanishes in the  $\xi_W = 1$  gauge while the other part split into  $\delta V$  is the same as in the Denner-Sack prescription. Thus the two prescriptions are essentially identical but now appear as an acceptable construction in that context. The difference to Eq. (11) is also clear: including in Eq. (11) the first term of Eq. (10) evaluated in the  $\xi_W = 1$  gauge (and its counterpart in the down-type sector) goes back to their prescriptions. Such scheme dependence seems unavoidable in CKM matrix renormalization. There seems to be no simple relations to the prescriptions suggested in Refs. [5, 7, 8] as they either employ subtraction at zero momentum or make reference to a theory with no mixing.

In practice it is often convenient to work with a specific representation of the CKM matrix although physical results are rephasing invariant and cannot depend on which representation we use. When doing so, we must be careful in interpreting  $\delta V$  in terms of counterterms for rotation angles and CP phase introduced in a given representation [13]. To see the point, we take a look at the UV divergent terms in Eq. (11) which are universal to all prescriptions suggested so far (in  $4 - 2\epsilon$  dimensions),

$$\left(\frac{3}{2\epsilon} \frac{\alpha}{16\pi s_W^2 m_W^2}\right)^{-1} \tilde{\delta V}_{\alpha i}^{\text{div}} = \sum_{\beta \neq \alpha} \sum_j V_{\beta i} V_{\alpha j} V_{\beta j}^* \frac{m_\beta^2 + m_\alpha^2}{m_\beta^2 - m_\alpha^2} m_j^2 + \sum_{j \neq i} \sum_\beta V_{\beta i} V_{\alpha j} V_{\beta j}^* \frac{m_j^2 + m_i^2}{m_j^2 - m_i^2} m_\beta^2. \quad (12)$$

We use  $\alpha, \beta, \gamma$  ( $i, j, k$ ) to distinguish the three up-type (down-type) quarks, and denote the rephasing invariant CP violating parameter as  $J = \text{Im}[V_{\alpha j} V_{\beta i} V_{\alpha i}^* V_{\beta j}^*]$ . Then,

$$\begin{aligned} & \left(\frac{3}{2\epsilon} \frac{\alpha}{16\pi s_W^2 m_W^2}\right)^{-1} \frac{|V_{\alpha i}|^2}{J} \text{Im} \left[ \frac{\tilde{\delta V}_{\alpha i}^{\text{div}}}{V_{\alpha i}} \right] \\ &= -2(m_\beta^2 - m_\gamma^2)(m_j^2 - m_k^2) \left[ \frac{m_\alpha^2}{(m_\beta^2 - m_\alpha^2)(m_\gamma^2 - m_\alpha^2)} + \frac{m_i^2}{(m_j^2 - m_i^2)(m_k^2 - m_i^2)} \right]. \end{aligned} \quad (13)$$

The above indicates clearly that an imaginary part will be induced for the element  $\tilde{\delta V}_{\alpha i}$  even if we start with a representation in which the element  $V_{\alpha i}$  is real. Namely, there is a rephasing effect due to renormalization which brings the CKM matrix from one representation before renormalization to another after. On the other hand, it is natural to require that a real bare element should have a real counterterm. Fortunately, this can be accommodated by employing the degrees of freedom in the on-shell renormalization scheme.



So far we have been focusing on the off-diagonal part in the anti-Hermitian combination of wave-function renormalization constants, but said nothing about the diagonal part shown in Eq. (9). Using field rephasing we can make five out of nine elements in the CKM matrix real while the remaining four will contain a representation dependent CP phase. This suggests the following modification to the CKM counterterm,

$$\delta V_{\alpha i} = \tilde{\delta} V_{\alpha i} + \frac{i}{2} V_{\alpha i} \operatorname{Im} \left[ -\delta U_{\alpha\alpha}^L + \delta D_{ii}^L \right], \quad (14)$$

which can be used to arrange that the bare and renormalized matrices are in the same representation. For example, in the original Kobayashi-Maskawa parametrization [1], the elements in the first row and column are real. We adjust the second terms in the above equation for  $(\alpha, i) = (1, 1), (1, 2), (1, 3), (2, 1), (3, 1)$  to absorb completely the imaginary part contained in  $\tilde{\delta} V_{\alpha i}$ 's so that the corresponding  $\delta V_{\alpha i}$ 's are real. This also fixes uniquely the second terms for the other four complex elements. The final result can be cast in a compact form applicable to all entries in the KM parametrization,

$$\frac{\delta V_{\alpha i}}{V_{\alpha i}} = \frac{\tilde{\delta} V_{\alpha i}}{V_{\alpha i}} - i \left[ \frac{\operatorname{Im} \tilde{\delta} V_{\alpha 1}}{V_{\alpha 1}} + \frac{\operatorname{Im} \tilde{\delta} V_{1i}}{V_{1i}} - \frac{\operatorname{Im} \tilde{\delta} V_{11}}{V_{11}} \right]. \quad (15)$$

Note that all nice features of  $\tilde{\delta} V_{\alpha i}$  carry over to  $\delta V_{\alpha i}$  since the above modification amounts to a  $\xi_W$  independent rephasing to the matrix and a reshuffling of terms in the renormalized amplitude that are cancelled anyway. But  $\delta V_{\alpha i}$  now allows for an interpretation in terms of rotation angles and CP phase.

### 3 Conclusion

We have shown in this work that a simple inspection of the quark self-energies suggests a prescription for the CKM matrix renormalization. The obtained matrix counterterm satisfies the unitarity constraint and is gauge parameter independent. It also leads to a physical amplitude which is UV finite, gauge parameter independent, and smooth in quark mass difference. We further improve it using the freedom in the on-shell renormalization scheme so that the counterterm also allows for an interpretation in terms of those of parameters in any given representation of the CKM matrix.

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- [13] It has been proposed in Ref. [8] to express directly the counterterm for the CKM matrix in terms of those of rotation angles and CP phase in a given representation. However, as explained here, this is not possible without removing a rephasing beforehand.